Grade 6 Math Circles

February 5th, 2024

Number Theory: Divisibility and Proofs - Problem Set

Note: Problems that are marked with * are considered challenge problems!

1. List all the positive whole numbers that are divisors of the following numbers;
   (a) 60
   (b) 180
2. List all the positive whole numbers that are divisors of 3, 9, 27, and 81. What do you notice?
3. Use the definition of divisibility to show that all numbers divide 0.
4. Use the Rule for Divisibility by 3 to determine if the following statements are true or false:
   (a) $3 \mid 81$
   (b) $3 \mid 1111111111111111111$
   (c) $3 \mid 222222222$
   (d) $3 \mid 1293746$
   (e) $3 \mid 343293$
5. Use the Rule for Divisibility by 4 to determine if the following statements are true or false:
   (a) $4 \mid 17$
   (b) $4 \mid 222222222$
   (c) $4 \mid 1293744$
   (d) $4 \mid 18318716$
   (e) $4 \mid 3432908$
6. Use the Rule for Divisibility by 5 to determine if the following statements are true or false:
   (a) $5 \mid 25$
   (b) $5 \mid 117365$
   (c) $5 \mid 1293744$
7. Use the Rule for Divisibility by 6 to determine if the following statements are true or false:
   (a) $6 \mid 24$
   (b) $6 \mid 1173657$

   (c) $6 \mid 1000100100$
   (d) $6 \mid 1293744$

8. Use the Rule for Divisibility by 7 to determine if the following statements are true or false:
   (a) $7 \mid 84$
   (b) $7 \mid 365$
   (c) $7 \mid 10000$
   (d) $7 \mid 11111$
   (e) $7 \mid 1293744$

9. Use the Rule for Divisibility by 8 to determine if the following statements are true or false:
   (a) $8 \mid 16$
   (b) $8 \mid 365$
   (c) $8 \mid 10000$
   (d) $8 \mid 11111$
   (e) $8 \mid 1293744$

10. Use the Rule for Divisibility by 9 to determine if the following statements are true or false:
    (a) $9 \mid 18$
    (b) $9 \mid 36582$
    (c) $9 \mid 10100100100101010100001000$
    (d) $9 \mid 11112$
    (e) $9 \mid 1293744$

11. Come up with rules for division by 18 and 24.

12. Use the rules of divisibility to fully factor 2520.

13. * Let x be a 4 digit number. Prove that **if** $9 \mid x$, **then** the digits of x add up to a multiple of 9.

14. ** Let x be 2 digit whole number. Prove that **if** $7 \mid x$, **then** the difference between $2 \times$ the ones digit of x and the remaining part of x is divisible by 7.

15. *** Let x be a six digit number given by $x = abcdef$. Show that **if** x is divisible by 101, **then** $(ab - cd + ef)$ is divisible by 101.